# Custom Approach to the Cost Estimation of the Full Truckload Contracts for Short Routes

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***Abstract*—Shipping of the goods is crucial for the development of the present economy. The transportation may be realized in many ways. This work focuses on the Full Truck Load (FTL) road transportation model. Such services are often realized using external fleet, and then there is a need for a tool that compares such offers, i.e. which allows to estimate the desired shipping cost. Generally, the FTLs fit to the long range routes. Estimation of such contracts is common and can be realized with different approaches, like calculators or sophisticated machine learning solutions. Apart of that, the need for the shipment cost estimation is also required for short and very short routes, which frequently support long routes. The rules for pricing of the FTL short routes differs from the long ones and thus the approaches used differ as well. This work presents custom approach specifically focused on that task. The assessment is performed using real multi-year contract data of several shipping companies operating in the European market.**

***Keywords*—cost estimation, full truck loads, machine learning, regression, clustering, DBSCAN**

## I. Introduction

Full truckload (FTL) constitutes a popular form of goods shipping with the goods filling an entire truck. It works well for large volumes with one load covering the whole truck. There exists an alternative method, i.e. the less than truckload (LTL), in which a truck transports partial loads to different locations within a single route. This paper focuses on the FTL approach, however from a rare and a very specific perspective.

This work considers the case of the external fleet contract pricing [1]. It assumes that the contractor uses its own dynamic pricing approach [2]. The challenge becomes more demanding in case of short routes, when the fundamental relationships with fuel costs and the driver work time have a secondary meaning. External fleet long range contracts pricing can be solved with the use of popular fright cost calculators [3], or with the use of artificial intelligence (AI) and machine learning (ML) methods [1, 4].

The short range external fleet FTL transportation cost estimation is not common in the literature. Actually, this aspect remains hidden in the overall task and researchers practically do not distinguish short routes as separate ones. Our observations during the realization of the project are clear, despite the modeling method, which we have tested. The biggest challenge in the external fleet FTL contracts cost estimation and the highest prediction errors appear for the short routes and low costs. The general absolute residuum measures might seem to be low, while the relative indexes remain suspiciously high due to the share of the low cost routes. Therefore, we have decided to look closer at this aspect and to decompose the problem into two subproblems. This work addresses the more challenging task of the short routes, while the longer ones are already covered [1]. The FTL cost estimation subject is described in Section II. The case study and used data are presented in Section III. This work proposes a specific approach presented in Section IV. Section V assesses the results, and the paper is concluded in Section VI.

## II. FTL Cost Estimation Models

The FTL freight cost estimation model is needed as external fleets mostly use dynamic pricing strategies [5]. Any additional information about potential cost sources helps, especially at the stage of model structure definition. Probably a combination of market and non-market factors might be useful [6]. Appropriate choice of the input features makes the estimation more reliable [7].

Contract dependent factors describe the type of the truck and the needed specific equipment, ADR (*l'Accord europèen relatif au transport international des marchandises Dangereuses* par *Route* – hazardous materials) or driver certificates. Pickup and unload locations determine the route, its length and estimated travel time. The loading location and its time must be coordinated with the actual drivers availability. The literature prefers the blind machine learning approaches [8–11] or less frequently complex hybrid ones [1]. Those blind ML approaches just take the data as it is and do evaluations. Once the data are wrong, falsified, or incomplete, the obtained model and its predictions would not work. We must remember that wrong data at the input give, wrong prediction at the output, no matter how sophisticated the model is.

## III. The Case Study

The data, which are used to assess the method originate from the real data of sample Polish road shipping companies [10]. Original dataset consists of some 414,000 records. The data, which are taken into account in the current research are limited only to the short range contracts, what limits the number of data to 18,895 records from the time period from January 1st, 2016 to April 30th, 2022. These record are used as the training dataset. Separate records starting from May 1st, 2022 until August 1st, 2022 are taken into consideration during validation. The dataset was subject to careful analysis and filtering, specifically targeting short-range freight originating from and destined for the same regional cluster [1]. Finally, validation set includes 619 (see Table 1).

Table 1. Number of data records

| | raw data | data after pre-processing |
|---|---|---|
| **training** | 414 404 | 18 895 |
| **validation** | 14 968 | 619 |

### *A. Data Preprocessing and Features Selection*

Each data record is explained by 22 inputs (independent variables). This number is limited to 12, the most important ones. This preprocessing is done using the process knowledge, shipping operators expertise and data analysis.

Furthermore, the initial data processing revealed the presence of missing data. A number of records within the database had missing values that potentially might affect the modeling and the training. It was observed that these voids occurred consistently across the dataset and could be divided into three major groups: missing values for freight distance, missing maximum trailer capacity and missing value for effective tonne-kilometer of the freight.

One has to remember that the analysis uses real shipping company database, which might have human errors, erroneous input values or just the missed ones.

A precise effort is done to address the recovery of missing data. Through an in-depth examination of the data and consultations with industry experts, three methods of data recovery are employed.

Firstly, it is observed that missing values for freight distance are a result of data export errors. Certain records display identical values for total distance traveled and distance traveled without load. Given that these records feature at least one recorded load and unload event, it would be implausible for the entire distance to be covered without a load. In such cases, it was assumed that the distance without load is considered to be zero.

Secondly, for instances where maximum trailer capacity information is missing, insights from an expert indicated that this resulted from a practice of using an empty field to indicate the utilization of the default 22-tonne trailer. Data are filled according to this information.

However, the recovery of missing values for tonne-kilometers proves to be the most perplexing. For the majority of records, this value is calculated as follows. Let $s_f$ represents the measured distance of the freight, and $m_f$ the measured weight of freight. The transportation work ($W_t$) can be determined by Eq. (1)

$$W_t = s_f \cdot m_f. \tag{1}$$

It is observed that a recovery method based on replacing the measured weight with the maximum capacity would not suffice, as not all transports utilized the trailer's full capacity. To address this fact, a straightforward approximation approach is used. First, the weight to capacity ratio $a_l$ is calculated across whole data set, for every filled data point $j$, as presented by Eq. (2)

$$a_l = \frac{1}{N_f} \sum_{j=1}^{N_f} \frac{W_t^j}{s_f^j \cdot c^j} \tag{2}$$

Then, the approximated transportation work is adjusted according to the calculated ratio. For every missing data point $i$, the transportation work is calculated as Eq. (3)

$$\widetilde{W_t^i} = a_l \cdot s_f^i \cdot c^i. \tag{3}$$

## IV. CUSTOM COST PREDICTION MODEL

The modeling consist of two main phases: structure identification (often called as the model design) and its parameters identification (denoted as model training).

### *A. Model Design*

The first step in model evaluation is to determine, which parameters should be included. The dataset contains x parameters, and to assess which of them significantly influenced the model's quality, a correlation analysis is run. For all parameters, the Pearson correlation coefficient between parameters and the actual cost value is calculated. In order to further expand the input data space, a kernel method is used. A series of base functions were proposed to transform the original parameters. For the newly obtained inputs, another correlation analysis is conducted.

Based on the correlation analysis, the model construction began. Initially, outputs were selected based on the highest Pearson correlation coefficient [12]. Then, additional inputs are added according to their correlation coefficient values. After each addition of a new input, its impact on the model's quality is examined. If it significantly affected the model's quality, the parameter is considered as important, and is added as a consecutive input variable. All inputs are presented in Table 2.

During the model creation process, a significant impact of the presence of a refrigeration unit during transportation was noticed. A considerable price difference was observed for transports with similar parameters but differing in the presence of a refrigeration unit. Due to this observation, it was decided to create separate models for transports with a refrigeration unit and those without. Let $w_i$ be the trainable model coefficients. The final form of the model equations is presented in Eq. (4).

$$\hat{y} = \begin{cases} \sum_{i=1}^{14} w_i \cdot x_i\,, & if\ x_{15} = 0 \\ \sum_{i=15}^{28} w_i \cdot x_{i-14}\,, & if\ x_{15} = 1 \end{cases} \tag{4}$$

Table 2. Model inputs

| Kernel symbol | Kernel description |
|---|---|
| $x_1$ | Free term (always *1*) |
| $x_2$ | Total freight distance |
| $x_3$ | Distance traveled with cargo |
| $x_4$ | Minimum estimated transit time |
| $x_5$ | Maximum estimated transit time |
| $x_6$ | Transportation lead time |
| $x_7$ | Transportation work – tonne-kilometers |
| $x_8$ | Number of loadings |
| $x_9$ | Number of unloadings |
| $x_{10}$ | Fuel price |
| $x_{11}$ | Time elapsed since transport completion |
| $x_{12}$ | Total freight distance minus distance traveled with cargo |
| $x_{13}$ | Total freight distance times fuel price |
| $x_{14}$ | Transportation lead time times fuel price |
| $x_{15}$ | Presence of refrigeration unit (*1* if present, *0* if not) |

### B. Model Training

The full original dataset is divided into clusters based on regional and transportation specifics using the DBSCAN algorithm [13], as described in [1]. Regional characteristics of the transported freight may include the state of roads, road tolls, long-term freight deals, and much more. The inability to identify all regional factors with high certainty leads to the search of an alternative, more reasonable solutions. In order to reflect these specifics, a cluster-based modeling approach is introduced.

In this approach, the dataset is split into groups, enabling the training of multiple models. This ensures that each model can capture regional characteristics without requiring explicit identification. After all, these specific characteristics is deeply embedded within the data. The division of the data is presented in Fig. 1 and Fig. 2.

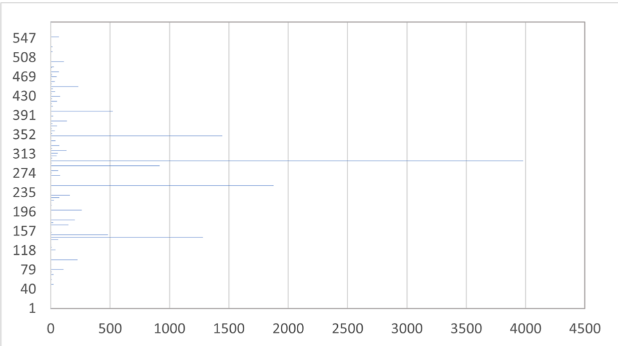


Fig. 1. Data points distribution among clusters.

The Fig. 2 reveals another issue – the training data is not uniformly distributed across the devised limits. From the presented clustering, three distinct classes of datasets emerge. These classes are denoted in this paper as: core, derived, and void clusters. Let $N_{th}$ represents the minimum number of data points necessary for a data to be considered feasible for learning purposes. Utilizing this criterion, we categorize the dataset clusters according to the following logics: clusters containing at least $N_{th}$ data points are classified as the core clusters. Clusters with fewer points than $N_{th}$, yet containing at least one point, are classified as the derived clusters. Finally, cluster datasets that do not contain any points are classified as void clusters. The design parameter value of $N_{th}$ is referred as the ‘training sufficiency threshold’.

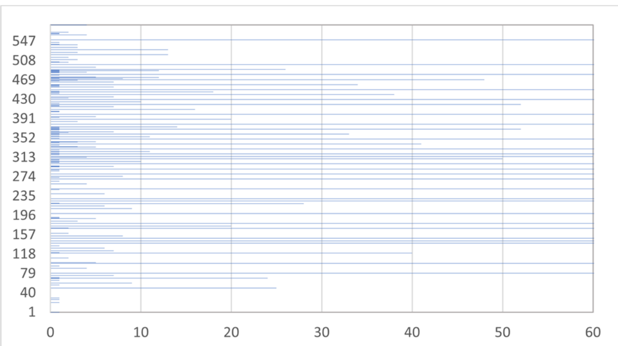


Fig. 2. Zoomed in data points distribution among clusters.

The training of cluster models is conducted in two stages. Initially, base cluster models ae trained using corresponding data sets as normal. Next, the derived models are trained using the 'data shuffling' technique. This method is used to increase the number of data points in sets containing less than $N_{th}$ samples. 'Data shuffling' is performed as follows:

1) Data points of a cluster to be trained are used to calculate the objective function output for every base model.
2) The best fitting model is chosen and the corresponding data points are introduced to the original data set creating a shuffled data set.
3) The derived cluster model is trained using shuffled data set and modified objective function.

Let assume that the $f(x)$ represents the original objective function, $p_s$ – a priority scalar and $x_b$, $x_d$ base and derived data points from the shuffled data set. Then, the shuffled objective function is represented by eq. (5)

$$f_s(x) = p_s \cdot f(x_d) + f(x_b). \tag{5}$$

The approach of data shuffling, as presented, aims at taking advantage of similarities between clusters and converge similar trajectory in different focal points – derived data samples.

Given the lack of data to train the void models, it is agreed to generate void model output with a simple algorithm. The proposed idea is to calculate outputs of every base and derived cluster model, and then use mean of results as an output of the void model. In practical applications, such predictions would be flagged as less certain (low quality value), as they wouldn’t be supported by the dedicated data.

Models are trained by minimizing the error between observed data and the model's output. The objective function is straightforward, taking the model's parameters as arguments and returning the error value for those parameters. For the purpose of comparison, two types of errors are used – mean square error (MSE) and mean absolute error (MAE) [14]:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x}_i)^2 \tag{6}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|x_i - \hat{x}_i| \tag{7}$$

To implement the training algorithm, the fmincon function from the Matlab Optimization Toolbox is used.

The last addition to the designed objective function stems from a practical challenge associated with the real-world application of the cost prediction model. From a practical standpoint, when a company employs the model to predict freight costs and uses these predictions to formulate prices for their clients, two scenarios must be taken into account.

In the first scenario, consider a situation where the model's prediction error is negative – indicating that the model overestimated the cost. In such a case, if the offered price is accepted, the error would lead to additional revenue for the company, as it anticipates higher freight costs.

On the contrary, in the second scenario, if the model's prediction error is positive, it may result in an unforeseen additional cost for the company. In extreme cases, this unexpected cost might even surpass the markup of the freight price. Based on the aforementioned scenarios, an asymmetry of error effects can be observed.

To address this asymmetry, the objective function is adjusted, leading to the introduction of the "underestimation penalty". The mechanism of underestimation penalty relies on discerning the sign of error, when calculating the objective function. Let ξ represents the underestimation penalty coefficient. Consequently, the objective function is defined by eqs. (8) and (9).

$$f_i(x) = \begin{cases} |x_i^* - x_i|, & if\ x_i^* - x_i \leq 0 \\ \xi \cdot |x_i^* - x_i|, & if\ x_i^* - x_i > 0 \end{cases} \tag{8}$$

$$f(x) = \sum_{i=1}^{N} f_i(x) \tag{9}$$

## V. RESULTS

In the following paragraphs results of the modeling is done. Actually, four different models are assessed:
mod1: performance index: MSE and $\xi=1.0$,
mod2: performance index: MSE and $\xi=1.5$,
mod3: performance index: MAE and $\xi=1.0$,
mod4: performance index: MAE and $\xi=1.5$.

The models are compared with different performance indexes. Three main integral indexes: mean square error (MSE), mean integral absolute error (IAE) and mean percentage integral absolute error (MAPE) are used [14]:

$$MAPE = \frac{100}{n}\sum_{i=1}^{n}\left|\frac{x_i - \hat{x}_i}{x_i}\right| \tag{10}$$

Apart of standard integral indexes, three statistical measures are also considered: normal standard deviation (stDev), the robust estimator of standard deviation in form of the logistic psi-function estimator [15] (RstDev) and the scale factor ($b$) of the Laplace distribution as in eq. (11), sometimes called as the double exponential probabilistic density function.

$$f_{\mu,b}(x) = \frac{1}{2b}e^{-\frac{|x-\mu|}{b}} \tag{11}$$

The residuum analysis begins with the evaluation of the selected performance indices. Table 3 shows values of the evaluated measures. Observation of their values reveals the fact that they have extremely large values except the one index – the robust standard deviation. This suggest that the residua include some extremely large values.

Table 3. Residuum analysis of the considered models for all clusters (green color highlights the best predictions according to the selected measure)

| | mod1 | mod2 | mod3 | mod4 |
|---|---|---|---|---|
| **MSE** | 2.134E+21 | **1.438E+21** | 2.054E+27 | 2.054E+27 |
| **MAE** | 3.216E+09 | **2.640E+09** | 3.155E+12 | 3.155E+12 |
| **MAPE** | 5.902E+07 | **4.845E+07** | 5.791E+10 | 5.791E+10 |
| **stDev** | 4.612E+10 | **3.786E+10** | 4.525E+13 | 4.525E+13 |
| **RstDev** | 83.45 | 83.06 | **65.43** | 79.39 |
| **scale *b*** | 6.400E+09 | **5.254E+09** | 6.280E+12 | 6.280E+12 |

The review shows that probably there are extremely wrong predictions. And its's the fact. They appear for the cluster that have no data as shown in the predictions dataset in Table 4. From one perspective it is obviously wrong and should never happen. However, it happens. The removal of such data without any interpretation or hesitation is an *adhockery*. Simple comparison of a single index might not reveal such a situation. Therefore, it's an occasion to show how to deal with such cases.

At first, we see that the case is detected through the comparison of the normal standard deviation and its robust counterpart [16, 17]. It is even better observed in the prediction error histogram and fitting of the considered distribution (normal, robust and Laplace) to it. Fig. 3 presents the histogram of the model mod3 prediction error with fitted distributions. We clearly see that only robust function works, as it is able to deal with outliers, even such the extreme ones. A comparison of obtained four robust distributions is shown in Fig. 4.

Table 4. Sample of the predictions dataset with highlighted in red color extreme outliers for cluster no 30

| group | cost | m1Cost | m2Cost | m3Cost | m4Cost |
|---|---|---|---|---|---|
| … | | | | | |
| 17 | 611.8 | 499.0 | 533.6 | 1034.6 | 1656.9 |
| 17 | 934.0 | 1112.7 | 1154.9 | 400.0 | 77.5 |
| 17 | 116.1 | 535.9 | 564.3 | 1038.4 | 1722.5 |
| 17 | 1075.9 | 971.4 | 1006.2 | 873.9 | 549.7 |
| 30 | 11903.2 | 6.63E+11 | 5.45E+11 | 6.51E+14 | 6.51E+14 |
| 30 | 2650.0 | 6.63E+11 | 5.45E+11 | 6.51E+14 | 6.51E+14 |
| 30 | 11203.0 | 6.63E+11 | 5.45E+11 | 6.51E+14 | 6.51E+14 |
| 34 | 6353.0 | 3570.8 | 9472.3 | 4577.2 | 4425.4 |
| … | | | | | |

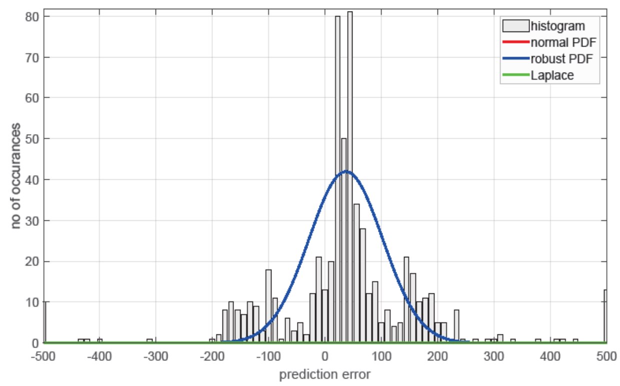


Fig. 3. Sample histogram plots with the fitted Laplace, normal and robust Gaussian distributions for the mod3.

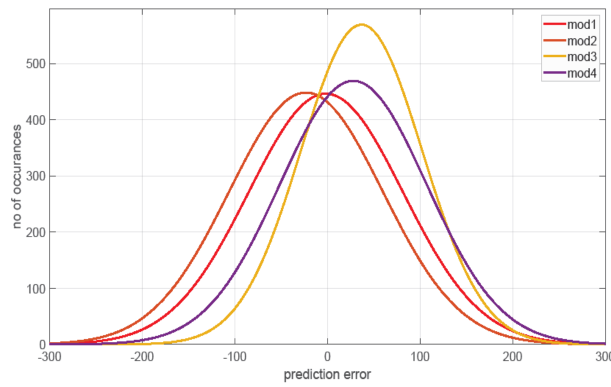


Fig. 4. Comparison of the robust Gaussian PDFs.

The review of the results points out that once the extreme outliers are considered the model mod2 is the best one. The robust index RstDev points out the model mod3. This observation is quite obvious as model mod2 is derived through the minimization of the MSE index, while mod3 is derived using the MAE index. We clearly see that outliers affect the prediction models assessment, especially the extreme ones. Finally, the quality of the models is compared using the box plot (see Fig. 5), which is independent on the data statistical properties and therefore objectively visualizes unspecified data.

Now, let see what happens once we remove those three extreme outliers. Table 5 presents the respective indexes. We clearly see that the values for indexes are comparable.

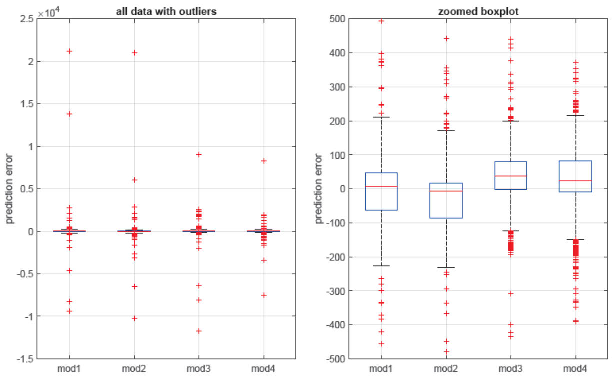

Fig. 5. Box plot models' comparison for all data.

Moreover, the model mod3 is indicated as the best one, which means that the outliers do not affect the assessment. The model mod1 is pointed out by MAPE index, which can be caused by the fact that mod1 gives better prediction for lower costs, contrary to the mod3.

Table 5. Residuum analysis of the considered models with rejected an extremely outlying cluster

| | mod1r | mod2r | mod3r | mod4r |
|---|---|---|---|---|
| **MSE** | 1400829 | 1108582 | **621617** | 989934 |
| **MAE** | 201.7 | 190.3 | **179.3** | 190.8 |
| **MAPE** | **35.56** | 37.63 | 37.10 | 39.31 |
| **stDev** | 1183.9 | 1053.7 | **788.5** | 995.8 |
| **RstDev** | 82.57 | 82.11 | **64.63** | 78.29 |
| **scale *b*** | 209.8 | 189.7 | **167.8** | 192.3 |

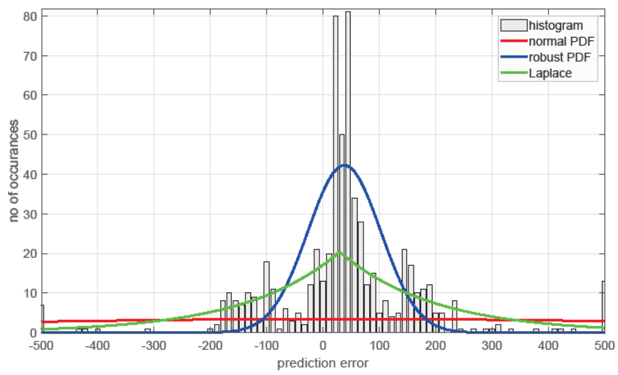

Fig. 6. Sample histogram plots with the fitted distributions for the mod34 (extreme outliers removed).

Next, the histograms are evaluated. Fig. 6 presents histogram plot for the best performing model mod3r, while Fig. 7 similar plot for the worst model mod1r. We see that gaussian distribution is significantly flat, what is caused by still existing and serious outlying residua. Laplace distribution is less affected by them, however only the robust version of the Gaussian distribution represents data properties to the highest possible extent.

The comparison of all four models is shown in Fig. 8, presenting all fitted robust distributions in a single plot. We can better observe the prediction error improvement. The model mod3r significantly outperforms the others.

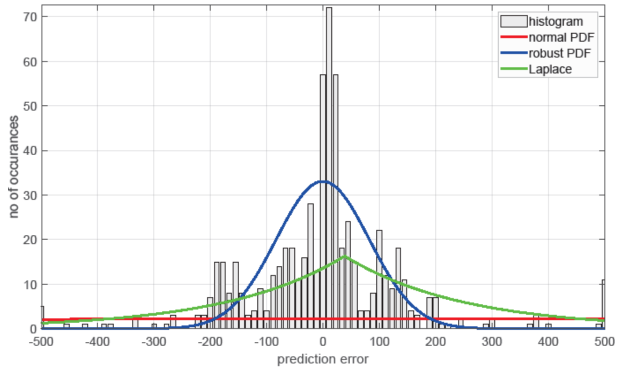

Fig. 7. Sample histogram plots with the fitted distributions for the mod1r (extreme outliers removed).

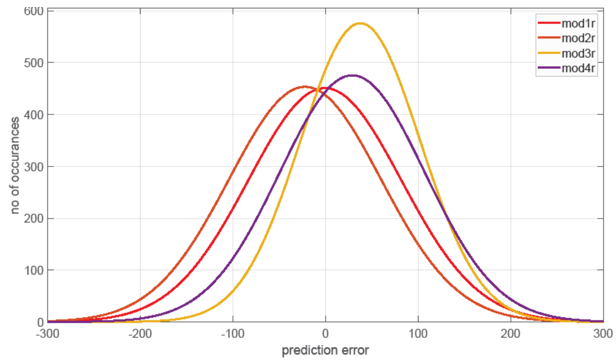

Fig. 8. Comparison of the robust Gaussian PDFs for the data without extreme outliers.

Analogously to the all data case, the Fig. 9 displays the boxplot for the case of data without extreme outliers. The very good thing is that the boxplots are very similar, what means that this form of the residuum analysis is not affected by the outliers, even the extreme ones.

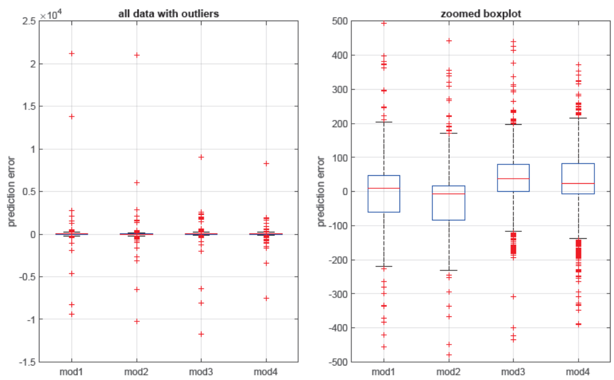

Fig. 1. Comparison of the models with the box plot for the data without extreme outliers.

This part of the analysis does not depend on the extreme outliers, as their position on the plots is far away of the diagrams borders and as they are not.

The comparison of the IAE and MAPE suggest that the estimation quality may be related to the shipping cost. One of the ways to detect it is to analyze the predicted versus the real costs relationship. Fig. 10 shows this plot for all four models.

The review of this plot allows to confirm several hypotheses. First, the majority of observations are for low costs with a few larger outliers. We also see, which models better work for these outlying observations. The extreme cost of 14000 exhibits the lowest residuum for the model mod3, while the mod1 model is the worst.

However the lowest costs are the most interesting from the process perspective, as short routs should be realized with low costs, while high values are human errors or rare, probably hybrid contracts (parts of intermodal routes).

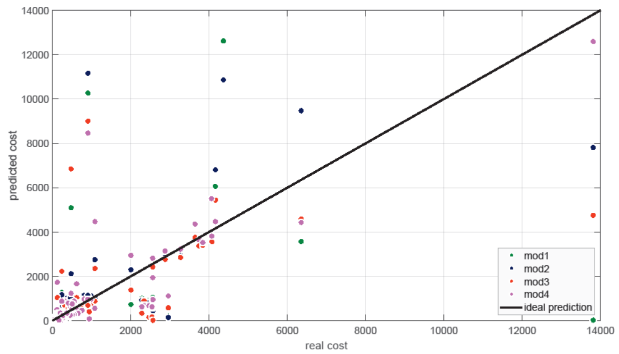


Fig. 10. The predicted versus real cost for all models

Fig. 11 presents the same plot, but focused on the low cost predictions. We observe the impact of the underestimation penalty coefficient $\xi$, as the predicted costs of mod2 are higher the those for mod1 and similar relationship is visible for mod3 and mod4.

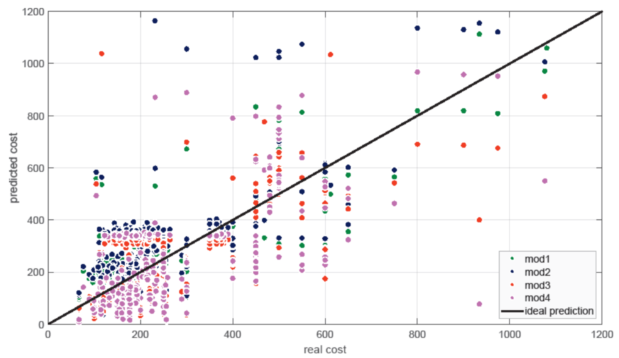


Fig. 11. The predicted versus real cost for all models – highlighted low costs

The dependance between an error and the route cost can be better visualized with the diagram shown in Fig. 12, which presents the relation between prediction error and the real shipping cost. This dependence is well visualized with the 3$^{rd}$ order polynomial fitting of the residuum to the real cost.

The interpretation of this relationship is disturbed by the outlying contracts. Fig. 13 highlights the prediction errors for route costs below 1200 PLN. This pictures confirms the effect of the underestimation penalty coefficient $\xi$.

Finally, Fig. 14 shows the same data but for o group of the lowest costs, less than 300 PLN. One should assume that this part of data should be the most representative for contracts. We observe clear relation prediction cost versus the real cost. And this part of contracts are predicted in a more reliable way than the others.

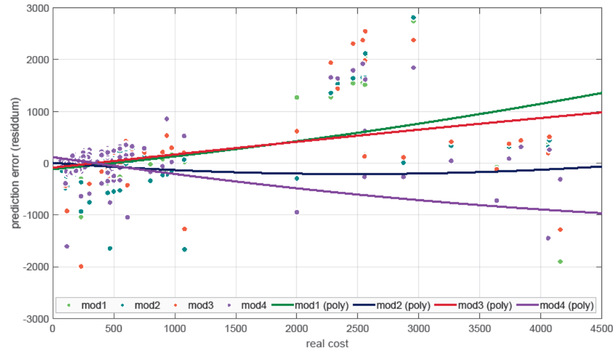


Fig. 12. The relationship between the model quality (prediction error) and the route shipping cost.

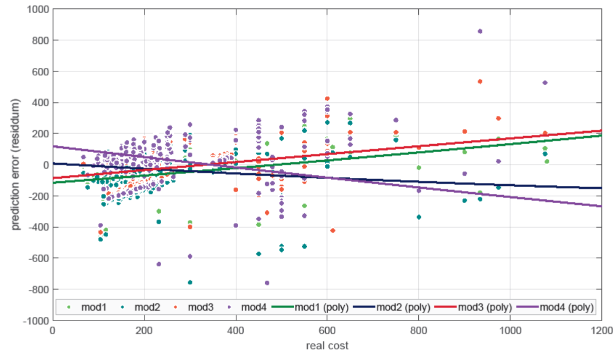


Fig. 13. The relationship between the model quality (prediction error) and the route shipping cost – highlighted low costs.

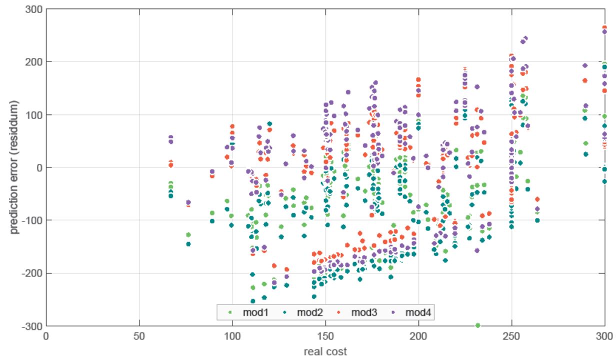


Fig. 14. The relationship between the model quality (prediction error) and the route shipping cost – highlighted the lowest costs.

Above analysis and stipulations show the subject of the short FTL routes predictions is more difficult than the long routes predictions [1].

As the subject is very specific, therefore the specific approach is used, as classical methods as the Extreme Gradient Boosting XGBoost [18] or k-NN (k Nearest Neighbors) estimator [19] are not sufficient.

One may feel dissatisfied due to the lack of comparison of the presented results with other solutions. The explanation for this fact has already been mentioned in the introduction. This task is not considered separately in the literature so far and it is difficult to refer to similar works.

Observation of the results enables not only to indicate prediction effectiveness. The fact that a particular estimators is better or worse according to some index is insufficient. It's worth to try to trace the reasons for such results using a root-cause analysis [20].

## V. Conclusions

This paper presents a proposal for the novel cost modeling approach for the FTL contracts in case of the dynamic pricing of the third-party transportation companies. The solution focuses on the short routes, which appear to be much more difficult and are seriously affected by the outliers, even the extreme ones. There are several reasons for these outlying data: human errors, interconnected contracts or a subscription model of pricing.

In the presented example, the non-Gaussian nature of the phenomenon and the distribution of errors is demonstrated, which should preclude the use of the MSE index or the Gaussian normal PDF. Shipping processes are associated with a lot of outliers resulting in fat tails. An this is a classic human behavior on data.

The subject demands further understanding of the fundamental process behind the process, deep understanding of human influence.

The analysis of the FTL shipping is an ongoing challenge. The deeper we dig into the subject, the more options appear. As usual, the further you go, the more complicated things become.

Customize solutions sometimes work, sometimes not. The intuition, which is behind them might be misleading. But still, we believe that it's better to use even the small trace of the information then use blind man black box approach.

Discussion of the method and its limitations suggest directions for further improvement. One perspective direction is to accommodate the approach using more specific understanding of the real shipping contracts behind the data. The next is to find the balance between relative and absolute errors. However, it should be remembered that this issue is universal and applies to the modeling task as such, regardless of the method used and the field of application.

## Conflict of Interest

The authors declare no conflict of interest.

## Author Contributions

JG and WS conducted the research; SC and PDD analyzed the data; JG, WS and PDD wrote the paper; PDD supervised the work; all authors had approved the final version.

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